$$x^2 + y^2 == 1$$
$$A\ (x - u)^2 + C\ (y - v)^2 == 1$$

$$x^2 + y^2 == 1$$

$$A\ (-u + x)^2 + C\ (-v + y)^2 == 1$$

Solve[$x^2 + y^2 == 1$, y **]**

$$\left\{\left\{y \to -\sqrt{1 - x^2}\right\},\ \left\{y \to \sqrt{1 - x^2}\right\}\right\}$$

Solve[$A\ (x - u)^2 + C\ (y - v)^2 == 1$, y **]**

$$\left\{\left\{y \to \frac{C\,v - \sqrt{C - A\,C\,u^2 + 2\,A\,C\,u\,x - A\,C\,x^2}}{C}\right\},\ \left\{y \to \frac{C\,v + \sqrt{C - A\,C\,u^2 + 2\,A\,C\,u\,x - A\,C\,x^2}}{C}\right\}\right\}$$

Path $y \to -\sqrt{1 - x^2}$

$$A\ (x - u)^2 + C\ (y - v)^2 == 1\ /.\ y \to -\sqrt{1 - x^2}$$

$$A\ (-u + x)^2 + C\ \left(-v - \sqrt{1 - x^2}\right)^2 == 1$$

$$\text{ExpandAll}\left[A\ (-u + x)^2 + C\ \left(-v - \sqrt{1 - x^2}\right)^2 == 1\right]$$

$$C + A\,u^2 + C\,v^2 - 2\,A\,u\,x + A\,x^2 - C\,x^2 + 2\,C\,v\,\sqrt{1 - x^2} == 1$$

$$C + A\,u^2 + C\,v^2 - 2\,A\,u\,x + A\,x^2 - C\,x^2 + 2\,C\,v\,\sqrt{1 - x^2} - 1 == 0$$

$$-1 + C + A\,u^2 + C\,v^2 - 2\,A\,u\,x + A\,x^2 - C\,x^2 + 2\,C\,v\,\sqrt{1 - x^2} == 0$$

$$-1 + C + A\,u^2 + C\,v^2 - 2\,A\,u\,x + A\,x^2 - C\,x^2 == -2\,C\,v\,\sqrt{1 - x^2}$$

$$-1 + C + A\,u^2 + C\,v^2 - 2\,A\,u\,x + A\,x^2 - C\,x^2 == -2\,C\,v\,\sqrt{1 - x^2}$$

$$\text{Collect}[-1 + C + A\,u^2 + C\,v^2 - 2\,A\,u\,x + A\,x^2 - C\,x^2,\ x] == -2\,C\,v\,\sqrt{1 - x^2}$$

$$-1 + C + A\,u^2 + C\,v^2 - 2\,A\,u\,x + (A - C)\ x^2 == -2\,C\,v\,\sqrt{1 - x^2}$$

$$-1 + C + A\,u^2 + C\,v^2 - 2\,A\,u\,x + (A - C)\ x^2 == -2\,C\,v\,\sqrt{1 - x^2}\ \ //.$$
$$\{A\,u \to R,\ C\,v \to Q,\ (A - C) \to M,\ -1 + C + A\,u^2 + C\,v^2 \to N\}$$

$$N - 2\,R\,x + M\,x^2 == -2\,Q\,\sqrt{1 - x^2}$$

$$\text{Expand}\left[\frac{1}{2\,Q}\ (N - 2\,R\,x + M\,x^2)\right] == -\sqrt{1 - x^2}$$

$$\frac{N}{2\,Q} - \frac{R\,x}{Q} + \frac{M\,x^2}{2\,Q} == -\sqrt{1 - x^2}$$

Path $y \to \sqrt{1 - x^2}$

$A\,(x - u)^2 + C\,(y - v)^2 == 1\ /.\ y \to \sqrt{1 - x^2}$

$A\,(-u + x)^2 + C\left(-v + \sqrt{1 - x^2}\right)^2 == 1$

$\text{ExpandAll}\left[A\,(-u + x)^2 + C\left(-v + \sqrt{1 - x^2}\right)^2 == 1\right]$

$C + A\,u^2 + C\,v^2 - 2\,A\,u\,x + A\,x^2 - C\,x^2 - 2\,C\,v\,\sqrt{1 - x^2} == 1$

$C + A\,u^2 + C\,v^2 - 2\,A\,u\,x + A\,x^2 - C\,x^2 - 2\,C\,v\,\sqrt{1 - x^2} - 1 == 0$

$-1 + C + A\,u^2 + C\,v^2 - 2\,A\,u\,x + A\,x^2 - C\,x^2 - 2\,C\,v\,\sqrt{1 - x^2} == 0$

$-1 + C + A\,u^2 + C\,v^2 - 2\,A\,u\,x + A\,x^2 - C\,x^2 == 2\,C\,v\,\sqrt{1 - x^2}$

$-1 + C + A\,u^2 + C\,v^2 - 2\,A\,u\,x + A\,x^2 - C\,x^2 == 2\,C\,v\,\sqrt{1 - x^2}$

$\text{Collect}[-1 + C + A\,u^2 + C\,v^2 - 2\,A\,u\,x + A\,x^2 - C\,x^2,\ x] == 2\,C\,v\,\sqrt{1 - x^2}$

$-1 + C + A\,u^2 + C\,v^2 - 2\,A\,u\,x + (A - C)\,x^2 == 2\,C\,v\,\sqrt{1 - x^2}$

$-1 + C + A\,u^2 + C\,v^2 - 2\,A\,u\,x + (A - C)\,x^2 == 2\,C\,v\,\sqrt{1 - x^2}\ //.$
$\{A\,u \to R,\ C\,v \to Q,\ (A - C) \to M,\ -1 + C + A\,u^2 + C\,v^2 \to N\}$

$N - 2\,R\,x + M\,x^2 == 2\,Q\,\sqrt{1 - x^2}$

$\text{Expand}\left[\frac{1}{2\,Q}\,(N - 2\,R\,x + M\,x^2)\right] == \sqrt{1 - x^2}$

$\frac{N}{2\,Q} - \frac{R\,x}{Q} + \frac{M\,x^2}{2\,Q} == \sqrt{1 - x^2}$

Part II

$\frac{N}{2\,Q} - \frac{R\,x}{Q} + \frac{M\,x^2}{2\,Q} == -\sqrt{1 - x^2}$

$\frac{N}{2\,Q} - \frac{R\,x}{Q} + \frac{M\,x^2}{2\,Q} == -\sqrt{1 - x^2}$

$\frac{N}{2\,Q} - \frac{R\,x}{Q} + \frac{M\,x^2}{2\,Q} == \sqrt{1 - x^2}$

$\{R \text{->} A\,u,$
$Q \text{->} C\,v,$
$M \text{->} (A - C),$
$N \text{->} -1 + C + A\,u^2 + C\,v^2\}$

$$\mathbf{ExpandAll}\left[\left(\frac{N}{2Q}-\frac{Rx}{Q}+\frac{Mx^2}{2Q}\right)^2 == 1-x^2\right]$$

$$\frac{N^2}{4Q^2}-\frac{NRx}{Q^2}+\frac{MNx^2}{2Q^2}+\frac{R^2x^2}{Q^2}-\frac{MRx^3}{Q^2}+\frac{M^2x^4}{4Q^2} == 1-x^2$$

$$\mathbf{Collect}\left[\frac{N^2}{4Q^2}-\frac{NRx}{Q^2}+\frac{MNx^2}{2Q^2}+\frac{R^2x^2}{Q^2}-\frac{MRx^3}{Q^2}+\frac{M^2x^4}{4Q^2}-(1-x^2),\ x\right] == 0$$

$$-1+\frac{N^2}{4Q^2}-\frac{NRx}{Q^2}+\left(1+\frac{MN}{2Q^2}+\frac{R^2}{Q^2}\right)x^2-\frac{MRx^3}{Q^2}+\frac{M^2x^4}{4Q^2} == 0$$

$$\mathbf{Collect}\left[\mathbf{Expand}\left[\frac{4Q^2}{M^2}\left(-1+\frac{N^2}{4Q^2}-\frac{NRx}{Q^2}+\left(1+\frac{MN}{2Q^2}+\frac{R^2}{Q^2}\right)x^2-\frac{MRx^3}{Q^2}+\frac{M^2x^4}{4Q^2}\right)\right],\ x\right] == 0$$

$$\frac{N^2}{M^2}-\frac{4Q^2}{M^2}-\frac{4NRx}{M^2}+\left(\frac{2N}{M}+\frac{4Q^2}{M^2}+\frac{4R^2}{M^2}\right)x^2-\frac{4Rx^3}{M}+x^4 == 0$$

$$\frac{N^2}{M^2}-\frac{4Q^2}{M^2}-\frac{4NRx}{M^2}+\left(\frac{2N}{M}+\frac{4Q^2}{M^2}+\frac{4R^2}{M^2}\right)x^2-\frac{4Rx^3}{M}+x^4 == 0\ //.$$
$$\left\{\frac{N^2}{M^2}-\frac{4Q^2}{M^2}\to d,\ -\frac{4NR}{M^2}\to c,\ \left(\frac{2N}{M}+\frac{4Q^2}{M^2}+\frac{4R^2}{M^2}\right)\to b,\ -\frac{4R}{M}\to a\right\}$$

$$d+cx+bx^2+ax^3+x^4 == 0$$

Quartic equation

$$\left\{a \text{ -> } -\frac{4R}{M},\quad b \text{ -> } \frac{2N}{M}+\frac{4Q^2}{M^2}+\frac{4R^2}{M^2},\quad c \text{ -> } -\frac{4NR}{M^2},\quad d \text{ -> } \frac{N^2}{M^2}-\frac{4Q^2}{M^2}\right\}$$

$$\mathbf{ExpandAll}\left[d+cx+bx^2+ax^3+x^4 == 0\ /.\ x \text{ -> } y-\frac{a}{4}\right]$$

$$-\frac{3a^4}{256}+\frac{a^2b}{16}-\frac{ac}{4}+d+\frac{a^3y}{8}-\frac{aby}{2}+cy-\frac{3a^2y^2}{8}+by^2+y^4 == 0$$

$$\mathbf{Collect}\left[-\frac{3a^4}{256}+\frac{a^2b}{16}-\frac{ac}{4}+d+\frac{a^3y}{8}-\frac{aby}{2}+cy-\frac{3a^2y^2}{8}+by^2+y^4,\ y\right] == 0$$

$$-\frac{3a^4}{256}+\frac{a^2b}{16}-\frac{ac}{4}+d+\left(\frac{a^3}{8}-\frac{ab}{2}+c\right)y+\left(-\frac{3a^2}{8}+b\right)y^2+y^4 == 0$$

$$-\frac{3a^4}{256}+\frac{a^2b}{16}-\frac{ac}{4}+d+\left(\frac{a^3}{8}-\frac{ab}{2}+c\right)y+\left(-\frac{3a^2}{8}+b\right)y^2+y^4 == 0\ //.$$
$$\left\{\left(-\frac{3a^2}{8}+b\right)\to e,\ \left(\frac{a^3}{8}-\frac{ab}{2}+c\right)\to f,\ -\frac{3a^4}{256}+\frac{a^2b}{16}-\frac{ac}{4}+d\to g\right\}$$

$$g+fy+ey^2+y^4 == 0$$

Quartic - part II

$$\text{FullSimplify}\left[\left(-\frac{3\,a^2}{8}+b\right)\text{ /. }\left\{a\text{ -> }-\frac{4\,R}{M},\ b\text{ -> }\frac{2\,N}{M}+\frac{4\,Q^2}{M^2}+\frac{4\,R^2}{M^2}\right\}\right]$$

$$\frac{2\,M\,N+4\,Q^2-2\,R^2}{M^2}$$

$$\text{FullSimplify}\left[\left(\frac{a^3}{8}-\frac{a\,b}{2}+c\right)\text{ /. }\left\{a\text{ -> }-\frac{4\,R}{M},\ b\text{ -> }\frac{2\,N}{M}+\frac{4\,Q^2}{M^2}+\frac{4\,R^2}{M^2},\ c\text{ -> }-\frac{4\,N\,R}{M^2}\right\}\right]$$

$$\frac{8\,Q^2\,R}{M^3}$$

$$\text{FullSimplify}\left[-\frac{3\,a^4}{256}+\frac{a^2\,b}{16}-\frac{a\,c}{4}+d\text{ /. }\right.$$
$$\left.\left\{a\text{ -> }-\frac{4\,R}{M},\ b\text{ -> }\frac{2\,N}{M}+\frac{4\,Q^2}{M^2}+\frac{4\,R^2}{M^2},\ c\text{ -> }-\frac{4\,N\,R}{M^2}\ ,\ d\text{ -> }\frac{N^2}{M^2}-\frac{4\,Q^2}{M^2}\right\}\right]$$

$$\frac{M^2\,(N^2-4\,Q^2)-2\,M\,N\,R^2+4\,Q^2\,R^2+R^4}{M^4}$$

$$g+f\,y+e\,y^2+y^4=0$$

$$g+f\,y+e\,y^2+y^4=0$$

Cubic equation

$$\left\{e\text{ -> }\frac{2\,M\,N+4\,Q^2-2\,R^2}{M^2},\right.$$
$$f\text{ -> }\frac{8\,Q^2\,R}{M^3},$$
$$\left.g\text{ -> }\frac{M^2\,(N^2-4\,Q^2)-2\,M\,N\,R^2+4\,Q^2\,R^2+R^4}{M^4}\right\}$$

$$x\text{ -> }y-\frac{a}{4}\text{ //. }\left\{a\text{ -> }-\frac{4\,R}{M}\right\}$$

$$x\to\frac{R}{M}+y$$

$$h^6+2\,e\,h^4+(e^2-4\,g)\,h^2-f^2=0\text{ /. }\{2\,e\to a_2,\ (e^2-4\,g)\to a_1,\ -f^2\to a_0\}$$

$$h^6+a_0+h^2\,a_1+h^4\,a_2=0$$

$$h^6+a_0+h^2\,a_1+h^4\,a_2=0\text{ /. }\{h^2\to s,\ h^4\to s^2,\ h^6\to s^3\}$$

$$s^3+a_0+s\,a_1+s^2\,a_2=0$$

$$\{a_0\text{ -> }-f^2,$$
$$a_1\text{ -> }e^2-4\,g,$$
$$a_2\text{ -> }2\,e\}$$

Cubic equation should be solved by using one of techniques available...

Original roots

$$\left\{\mathbf{R} \to \mathbf{A\,u},\ \mathbf{Q} \to \mathbf{C\,v},\ \mathbf{M} \to (\mathbf{A}-\mathbf{C}),\ \mathbf{N} \to -1+\mathbf{C}+\mathbf{A\,u}^2+\mathbf{C\,v}^2,\ \mathbf{e} \to \frac{2\,\mathbf{M\,N}+4\,\mathbf{Q}^2-2\,\mathbf{R}^2}{\mathbf{M}^2},\ \mathbf{f} \to \frac{8\,\mathbf{Q}^2\,\mathbf{R}}{\mathbf{M}^3},\ \mathbf{g} \to \frac{\mathbf{M}^2\,(\mathbf{N}^2-4\,\mathbf{Q}^2)-2\,\mathbf{M\,N\,R}^2+4\,\mathbf{Q}^2\,\mathbf{R}^2+\mathbf{R}^4}{\mathbf{M}^4}\right\};$$

$$\mathbf{x} \to \mathbf{y}-\frac{\mathbf{a}}{4}\ /.\ \left\{\mathbf{a} \to -\frac{4\,\mathbf{R}}{\mathbf{M}}\right\}$$

$$x \to \frac{R}{M}+y$$

$$\mathbf{h}^2 \to \mathbf{s}$$

$$h^2 \to s$$

$$\mathbf{Solve}[\mathbf{h}^2 == \mathbf{s},\ \mathbf{h}]$$

$$\{\{h \to -\sqrt{s}\},\ \{h \to \sqrt{s}\}\}$$

$$\mathbf{W}_1 \to \sqrt{\mathbf{s}-4\,\mathbf{j}}\ /.\ \mathbf{j} \to \frac{\mathbf{e}+\mathbf{h}^2-\frac{\mathbf{f}}{\mathbf{h}}}{2}$$

$$W_1 \to \sqrt{-2\left(e-\frac{f}{h}+h^2\right)+s}$$

$$\mathbf{W}_2 \to \sqrt{\mathbf{s}-\frac{4\,\mathbf{g}}{\mathbf{j}}}\ /.\ \mathbf{j} \to \frac{\mathbf{e}+\mathbf{h}^2-\frac{\mathbf{f}}{\mathbf{h}}}{2}$$

$$W_2 \to \sqrt{-\frac{8\,g}{e-\frac{f}{h}+h^2}+s}$$

$$\mathbf{x}_1 \to \frac{\frac{(-\mathbf{h}+\mathbf{W}_1)}{2}+\mathbf{R}}{\mathbf{M}}$$

$$\mathbf{x}_2 \to \frac{\frac{(-\mathbf{h}-\mathbf{W}_1)}{2}+\mathbf{R}}{\mathbf{M}}$$

$$\mathbf{x}_3 \to \frac{\frac{(\mathbf{h}+\mathbf{W}_2)}{2}+\mathbf{R}}{\mathbf{M}}$$

$$\mathbf{x}_4 \to \frac{\frac{(\mathbf{h}-\mathbf{W}_2)}{2}+\mathbf{R}}{\mathbf{M}}$$

Special cases

■ |M|>1

$$g + f\,y + e\,y^2 + y^4 == 0$$

$$e \text{ -> } \frac{2\,M\,N + 4\,Q^2 - 2\,R^2}{M^2},$$
$$f \text{ -> } \frac{8\,Q^2\,R}{M^3},$$
$$g \text{ -> } \frac{M^2\,(N^2 - 4\,Q^2) - 2\,M\,N\,R^2 + 4\,Q^2\,R^2 + R^4}{M^4}$$

$$g + f\,y + e\,y^2 + y^4 == 0 \text{ /. } \left\{e \text{ -> } \frac{2\,M\,N + 4\,Q^2 - 2\,R^2}{M^2},\ f \text{ -> } \frac{8\,Q^2\,R}{M^3},\ g \text{ -> } \frac{M^2\,(N^2 - 4\,Q^2) - 2\,M\,N\,R^2 + 4\,Q^2\,R^2 + R^4}{M^4},\ y \to \frac{t}{M}\right\}$$

Out[8]= $$\frac{M^2\,(N^2 - 4\,Q^2) - 2\,M\,N\,R^2 + 4\,Q^2\,R^2 + R^4}{M^4} + \frac{8\,Q^2\,R\,t}{M^4} + \frac{(2\,M\,N + 4\,Q^2 - 2\,R^2)\,t^2}{M^4} + \frac{t^4}{M^4} == 0$$

In[9]:= $$x \to \frac{R}{M} + y \text{ /. } y \to \frac{t}{M}$$

Out[9]= $$x \to \frac{R}{M} + \frac{t}{M}$$

■ u==0 and v==0

$$x^2 + y^2 == 1$$
$$A\,(x - u)^2 + C\,(y - v)^2 == 1 \text{ /. } \{u \to 0,\ v \to 0\}$$

$$x^2 + y^2 == 1$$

$$A\,x^2 + C\,y^2 == 1$$

```
Solve[ExpandAll[A x^2 + C y^2 == 1 /. y^2 → 1 - x^2], x]
```

$$\left\{\left\{x \to -\frac{\sqrt{1 - C}}{\sqrt{A - C}}\right\},\ \left\{x \to \frac{\sqrt{1 - C}}{\sqrt{A - C}}\right\}\right\}$$

■ u==0

$$x^2 + y^2 == 1$$
$$A\,(x - u)^2 + C\,(y - v)^2 == 1 \text{ /. } u \to 0$$

$$x^2 + y^2 == 1$$

$$A\,x^2 + C\,(-v + y)^2 == 1$$

```
Solve[x^2 + y^2 == 1, y]
```

$$\left\{\left\{y \to -\sqrt{1 - x^2}\right\},\ \left\{y \to \sqrt{1 - x^2}\right\}\right\}$$

ExpandAll$\left[-1 + A\,x^2 + C\,(-v + y)^2 == 0\ /.\ y \to -\sqrt{1 - x^2}\,\right]$

$-1 + C + C\,v^2 + A\,x^2 - C\,x^2 + 2\,C\,v\,\sqrt{1 - x^2} == 0$

$-1 + C + C\,v^2 + (A - C)\,x^2 + 2\,C\,v\,\sqrt{1 - x^2} == 0\ //.\ \{C\,v \to Q,\ (A - C) \to M,\ -1 + C + C\,v^2 \to N\}$

$N + M\,x^2 + 2\,Q\,\sqrt{1 - x^2} == 0$

ExpandAll$\left[(N + M\,x^2)^2 == \left(-2\,Q\,\sqrt{1 - x^2}\right)^2\right]$

$N^2 + 2\,M\,N\,x^2 + M^2\,x^4 == 4\,Q^2 - 4\,Q^2\,x^2$

Collect$[N^2 + 2\,M\,N\,x^2 + M^2\,x^4 - 4\,Q^2 + 4\,Q^2\,x^2,\ x] == 0$

$N^2 - 4\,Q^2 + (2\,M\,N + 4\,Q^2)\,x^2 + M^2\,x^4 == 0$

$N^2 - 4\,Q^2 + (2\,M\,N + 4\,Q^2)\,x^2 + M^2\,x^4 == 0\ //.$
$\{N^2 - 4\,Q^2 \to c,\ (2\,M\,N + 4\,Q^2) \to b,\ M^2 \to a,\ x^2 \to s,\ x^4 \to s^2\}$

$c + b\,s + a\,s^2 == 0$

Solve$[c + b\,s + a\,s^2 == 0,\ s]$

$\left\{\left\{s \to \frac{-b - \sqrt{b^2 - 4\,a\,c}}{2\,a}\right\},\ \left\{s \to \frac{-b + \sqrt{b^2 - 4\,a\,c}}{2\,a}\right\}\right\}$

$\left\{\left\{s \to \frac{-b - \sqrt{b^2 - 4\,a\,c}}{2\,a}\right\},\ \left\{s \to \frac{-b + \sqrt{b^2 - 4\,a\,c}}{2\,a}\right\}\right\}\ /.\ b^2 - 4\,a\,c \to D_1$

$\left\{\left\{s \to \frac{-b - \sqrt{D_1}}{2\,a}\right\},\ \left\{s \to \frac{-b + \sqrt{D_1}}{2\,a}\right\}\right\}$

FullSimplify$[D_1 \text{ -> } \textbf{ExpandAll}[b^2 - 4\,a\,c\ /.\ \{a \text{ -> } M^2,\ b \text{ -> } (2\,M\,N + 4\,Q^2),\ c \text{ -> } N^2 - 4\,Q^2\}]]$

$D_1 \to 16\,Q^2\,(M\,(M + N) + Q^2)$

PowerExpand$\left[\left\{\left\{s \to \frac{-b - \sqrt{D_1}}{2\,a}\right\},\ \left\{s \to \frac{-b + \sqrt{D_1}}{2\,a}\right\}\right\}\ /.\right.$
$\left.\{D_1 \to 16\,Q^2\,(M\,(M + N) + Q^2),\ a \text{ -> } M^2,\ b \text{ -> } (2\,M\,N + 4\,Q^2)\}\right]$

$\left\{\left\{s \to \frac{-2\,M\,N - 4\,Q^2 - 4\,Q\,\sqrt{M\,(M + N) + Q^2}}{2\,M^2}\right\},\ \left\{s \to \frac{-2\,M\,N - 4\,Q^2 + 4\,Q\,\sqrt{M\,(M + N) + Q^2}}{2\,M^2}\right\}\right\}$

$\left\{\left\{s \to \frac{-2\,M\,N - 4\,Q^2 - 4\,Q\,\sqrt{M\,(M + N) + Q^2}}{2\,M^2}\right\},\right.$
$\left.\left\{s \to \frac{-2\,M\,N - 4\,Q^2 + 4\,Q\,\sqrt{M\,(M + N) + Q^2}}{2\,M^2}\right\}\right\}\ /.\ M\,(M + N) + Q^2 \to D$

$\left\{\left\{s \to \frac{-2\,M\,N - 4\,\sqrt{D}\,Q - 4\,Q^2}{2\,M^2}\right\},\ \left\{s \to \frac{-2\,M\,N + 4\,\sqrt{D}\,Q - 4\,Q^2}{2\,M^2}\right\}\right\}$

PowerExpand$\left[x \to \sqrt{s}\ /.\ \left\{\left\{s \to \frac{-2\,M\,N - 4\,\sqrt{D}\,Q - 4\,Q^2}{2\,M^2}\right\},\ \left\{s \to \frac{-2\,M\,N + 4\,\sqrt{D}\,Q - 4\,Q^2}{2\,M^2}\right\}\right\}\right]$

$\left\{x \to \frac{\sqrt{-2\,M\,N - 4\,\sqrt{D}\,Q - 4\,Q^2}}{\sqrt{2}\,M},\ x \to \frac{\sqrt{-2\,M\,N + 4\,\sqrt{D}\,Q - 4\,Q^2}}{\sqrt{2}\,M}\right\}$

```
{Q -> C v,
 M -> (A - C),
 N -> -1 + C + C v^2,
 D -> M (M + N) + Q^2}
```

■ v==0

$$x^2 + y^2 == 1$$
$$A\ (x - u)^2 + C\ (y - v)^2 == 1\ /.\ \{v \to 0\}$$

$$x^2 + y^2 == 1$$

$$A\ (-u + x)^2 + C\ y^2 == 1$$

$$\text{Solve}[A\ (-u + x)^2 + C\ y^2 == 1\ /.\ y^2 \to 1 - x^2,\ x]$$

$$\left\{\left\{x \to \frac{A\,u - \sqrt{A - C - A\,C + C^2 + A\,C\,u^2}}{A - C}\right\},\ \left\{x \to \frac{A\,u + \sqrt{A - C - A\,C + C^2 + A\,C\,u^2}}{A - C}\right\}\right\}$$

■ A==C

$$\frac{N}{2\,Q} - \frac{R\,x}{Q} + \frac{M\,x^2}{2\,Q} == \sqrt{1 - x^2}$$

```
{R -> A u,
 Q -> C v,
 M -> (A - C),
 N -> -1 + C + A u^2 + C v^2}
```

$$M \text{ -> } (A - C)\ /.\ A \to C$$

$$M \to 0$$

$$\frac{N}{2\,Q} - \frac{R\,x}{Q} + \frac{M\,x^2}{2\,Q} == \sqrt{1 - x^2}\ /.\ M \to 0$$

$$\frac{N}{2\,Q} - \frac{R\,x}{Q} == \sqrt{1 - x^2}$$

$$\text{ExpandAll}\left[\left(\frac{N}{2\,Q} - \frac{R\,x}{Q}\right)^2 == 1 - x^2\right]$$

$$\frac{N^2}{4\,Q^2} - \frac{N\,R\,x}{Q^2} + \frac{R^2\,x^2}{Q^2} == 1 - x^2$$

$$\text{Solve}\left[\frac{N^2}{4\,Q^2} - \frac{N\,R\,x}{Q^2} + \frac{R^2\,x^2}{Q^2} == 1 - x^2,\ x\right]$$

$$\left\{\left\{x \to \frac{N\,R - \sqrt{-N^2\,Q^2 + 4\,Q^4 + 4\,Q^2\,R^2}}{2\,(Q^2 + R^2)}\right\},\ \left\{x \to \frac{N\,R + \sqrt{-N^2\,Q^2 + 4\,Q^4 + 4\,Q^2\,R^2}}{2\,(Q^2 + R^2)}\right\}\right\}$$

$$\left\{\left\{x \to \frac{N R - \sqrt{-N^2 Q^2 + 4 Q^4 + 4 Q^2 R^2}}{2 (Q^2 + R^2)}\right\}, \left\{x \to \frac{N R + \sqrt{-N^2 Q^2 + 4 Q^4 + 4 Q^2 R^2}}{2 (Q^2 + R^2)}\right\}\right\} \text{ //. }$$

$$\left\{-N^2 Q^2 + 4 Q^4 + 4 Q^2 R^2 \to D,\ \frac{1}{2 (Q^2 + R^2)} \to \frac{1}{F}\right\}$$

$$\left\{\left\{x \to \frac{-\sqrt{D} + N R}{F}\right\}, \left\{x \to \frac{\sqrt{D} + N R}{F}\right\}\right\}$$

■ A==C and u==0 and v==0 (F==0)

$x^2 + y^2 == 1$

$A (x - u)^2 + C (y - v)^2 == 1$ /. {A → C, u → 0, v → 0}

$x^2 + y^2 == 1$

$C x^2 + C y^2 == 1$

$y^2 == 1 - x^2$

$C x^2 + C y^2 == 1$ /. y^2 -> $1 - x^2$

$C x^2 + C (1 - x^2) == 1$

Solve[$C x^2 + C (1 - x^2) == 1$, x]

{}